1-6 Trig Functions

Learning Objectives:

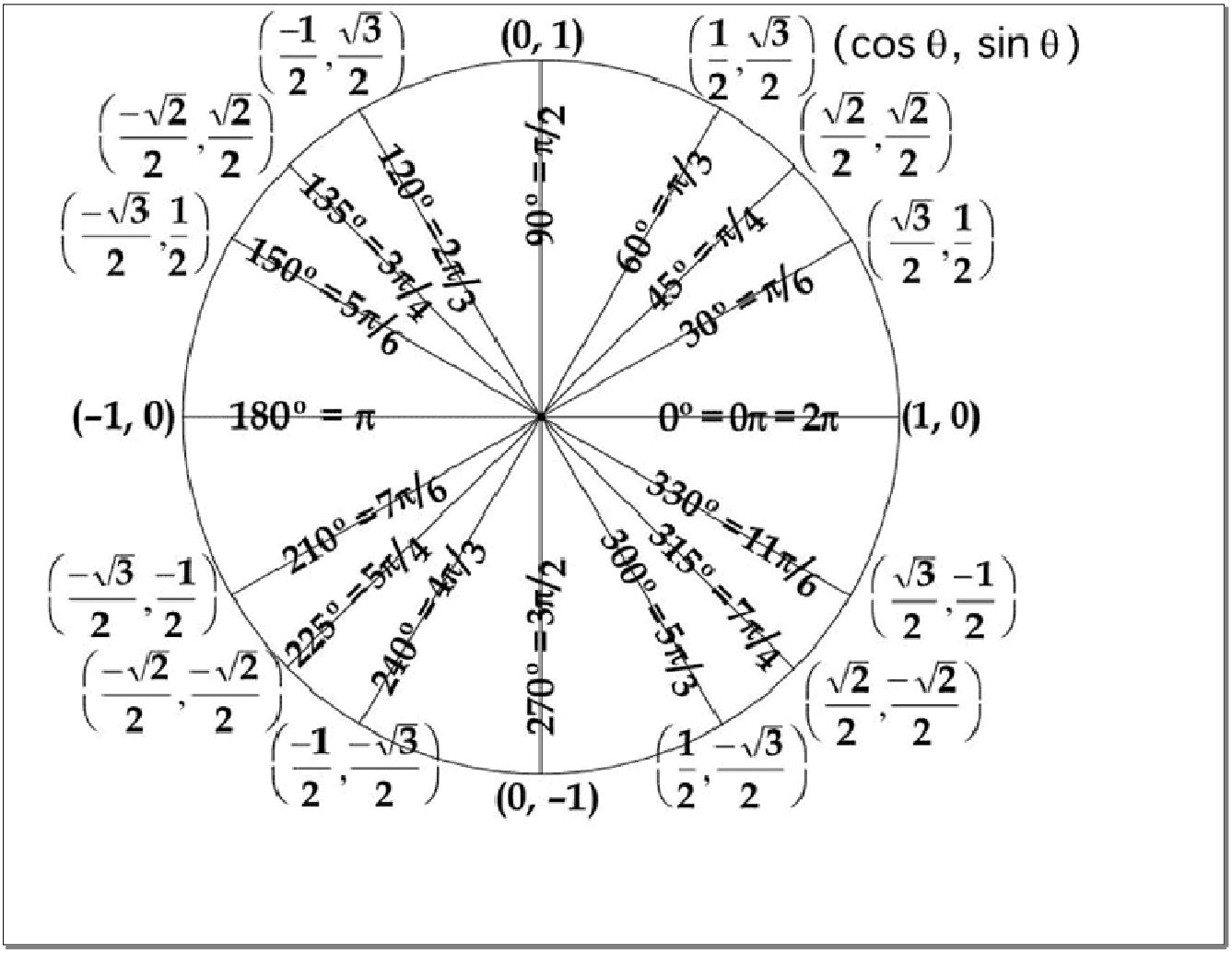
I can create a unit circle and use it to calculate the exact values of any of the 6 trig functions at any of the special angles.

I can graph any of the trig functions without the use of a graphing calculator.

I can solve trig equations.

I understand the relationship between any trig function and its inverse.

Sep 7-1:28 PM


(cos θ, sin θ)
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
90° = π/2
60° = π/3
45° = π/4
30° = π/6
0° = 0π = 2π
180° = π
120° = 2π/3
135° = 3π/4
150° = 5π/6
210° = 7π/6
225° = 5π/4
240° = 4π/3
270° = 3π/2
300° = 5π/3
315° = 7π/4
330° = 11π/6
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

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Ex1. Evaluate w/o using a GC

1.) $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

2.) $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

3.) $\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$

4.) $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$

5.) $\cos\left(\frac{23\pi}{3}\right) = \frac{1}{2}$

6.) $\sec\left(\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$ $\quad -\frac{2}{\sqrt{3}}$

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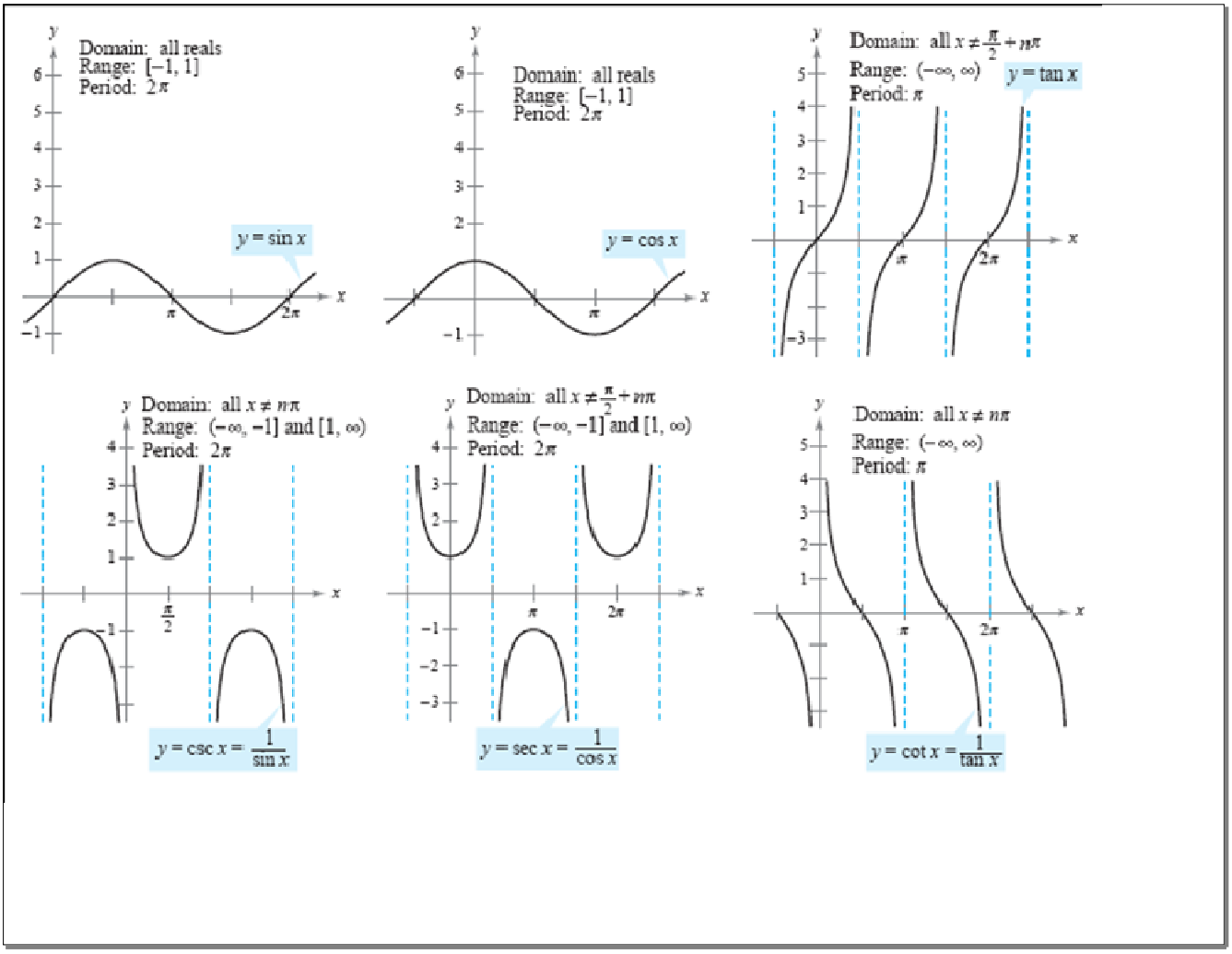
Domain: all reals
Range: [−1, 1]
Period: 2π
y = sin x
Domain: all reals
Range: [−1, 1]
Period: 2π
y = cos x
Domain: all x ≠ π/2 + nπ
Range: (−∞, ∞)
Period: π
y = tan x
Domain: all x ≠ nπ
Range: (−∞, −1] and [1, ∞)
Period: 2π
y = csc x = 1/sin x
Domain: all x ≠ π/2 + nπ
Range: (−∞, −1] and [1, ∞)
Period: 2π
y = sec x = 1/cos x
Domain: all x ≠ nπ
Range: (−∞, ∞)
Period: π
y = cot x = 1/tan x

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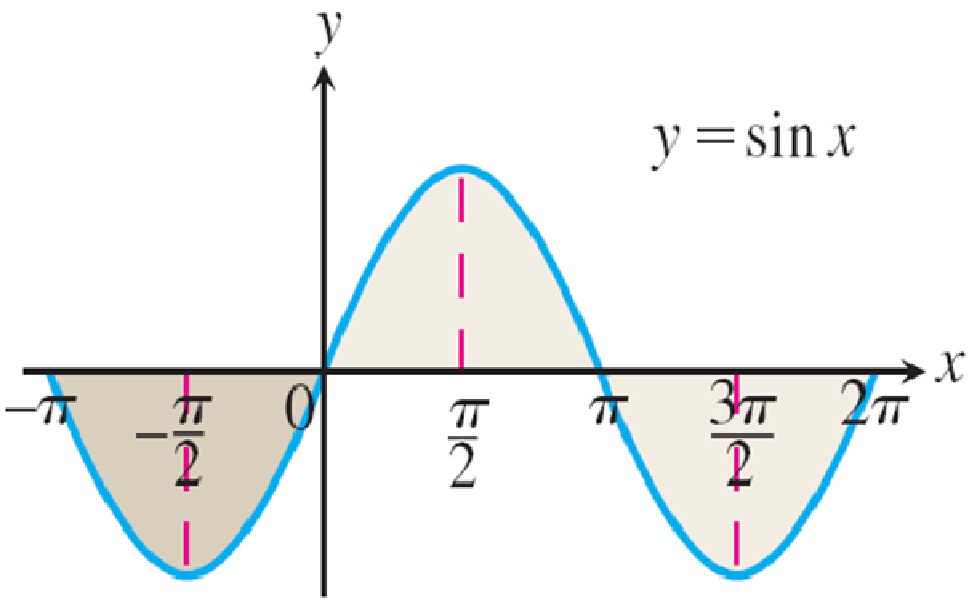

Domain: $-\infty < x < \infty$

Range: $-1 \le y \le 1$

Period: 2π

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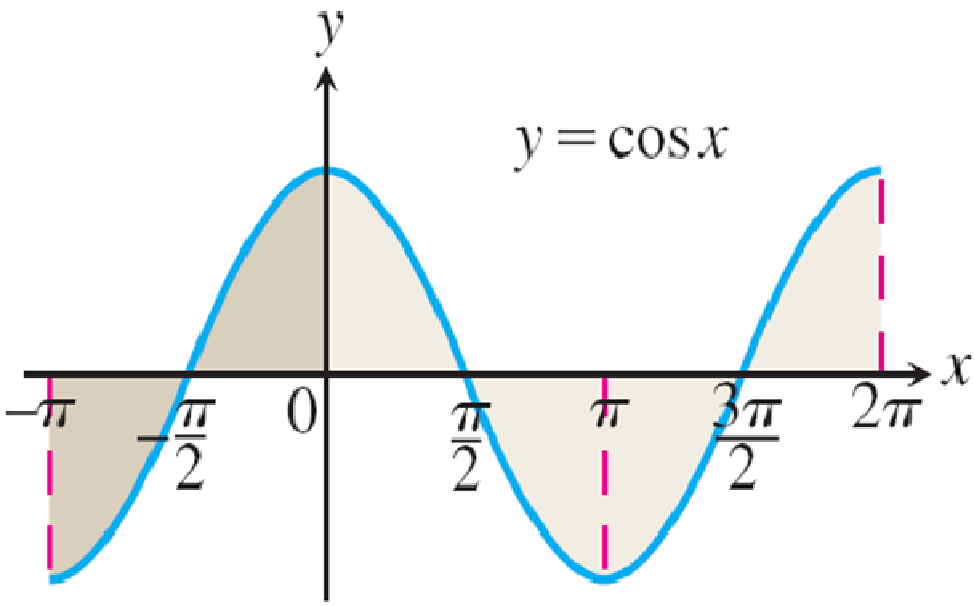

Domain: $-\infty < x < \infty$

Range: $-1 \le y \le 1$

Period: 2π

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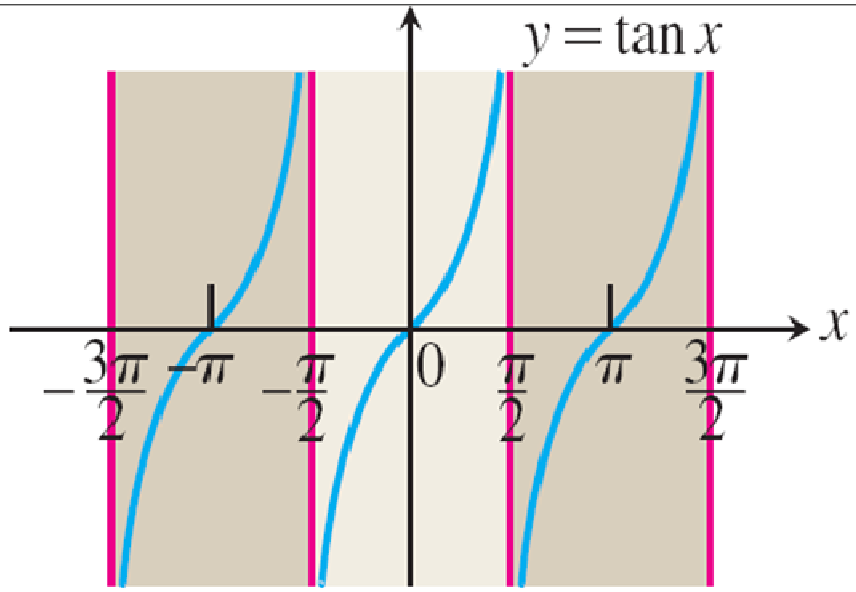

Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

Range: $-\infty < y < \infty$

Period: π

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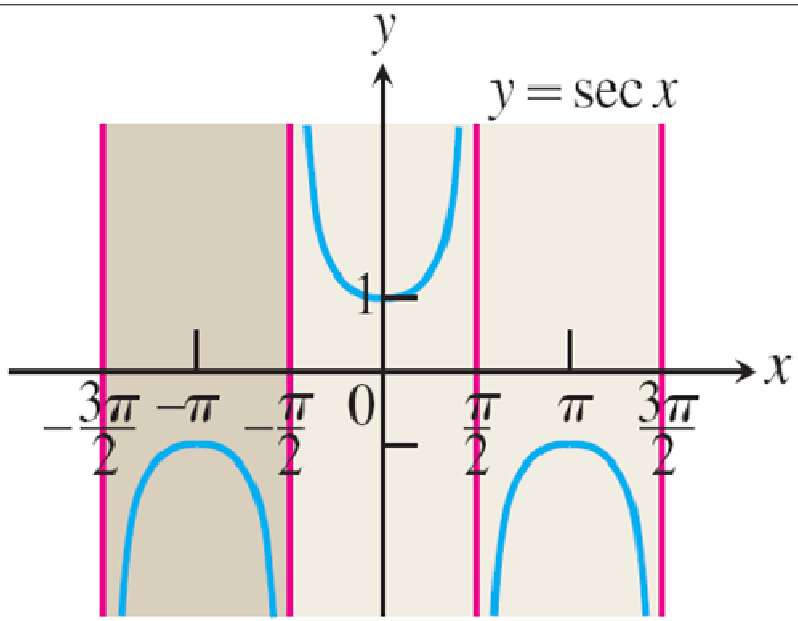

Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

Range: $y \leq -1$ and $y \geq 1$

Period: 2π

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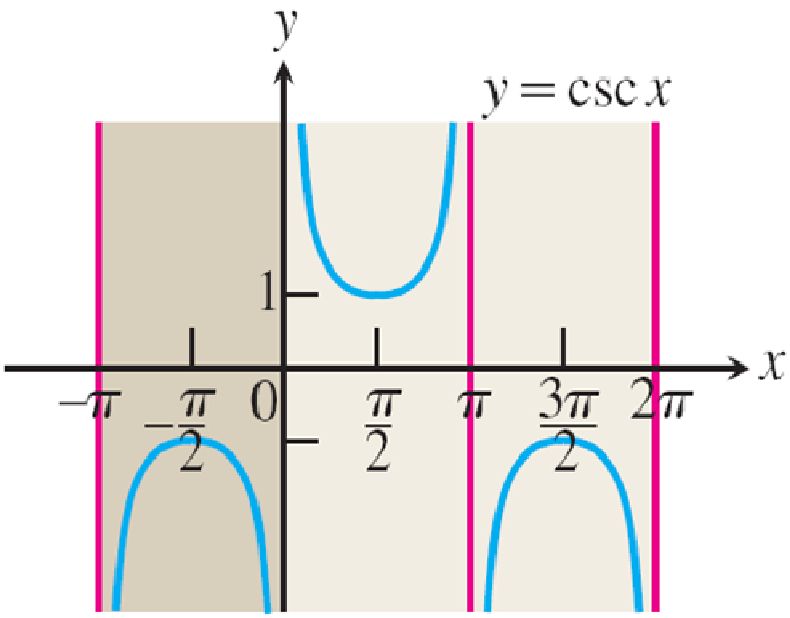

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$

Range: $y \leq -1$ and $y \geq 1$

Period: 2π

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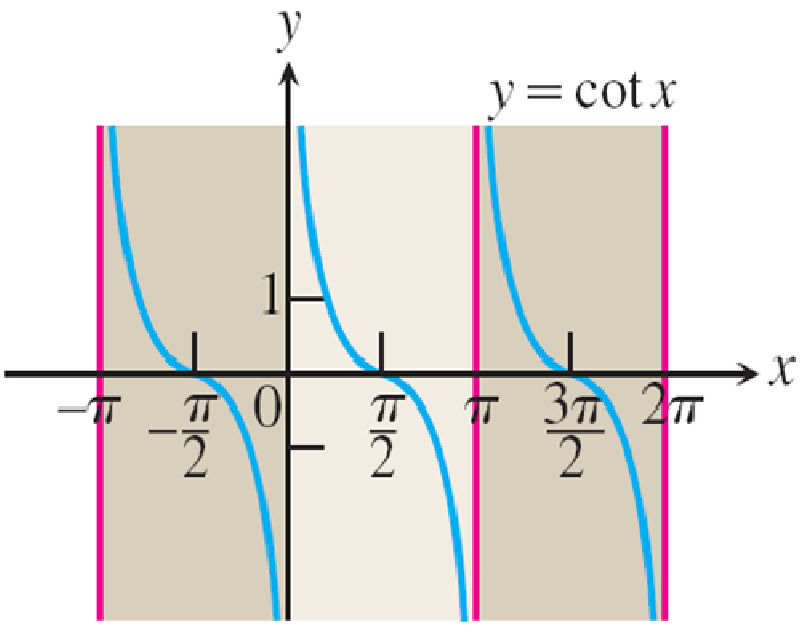

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$

Range: $-\infty < y < \infty$

Period: π

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Graphing Shifting Trig Functions

$$y = A\sin(k \cdot x + c) + h$$

Amplitude = A

Period = $\frac{2\pi}{k}$ for sin, cos, sec,and csc

$\frac{\pi}{k}$ for tan, cot

Vertical Shift = h + = up - = down

Horizontal Shift = $\frac{c}{k}$ + = left - = right

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Ex1. Graph (w/o a G.C.)

1.) $y = 3\sin\left(2x + \frac{\pi}{2}\right) - 1$

2.) $y = 2\sec\left(3x + \frac{\pi}{4}\right)$

3.) $y = \tan(4x - \pi) + 1$

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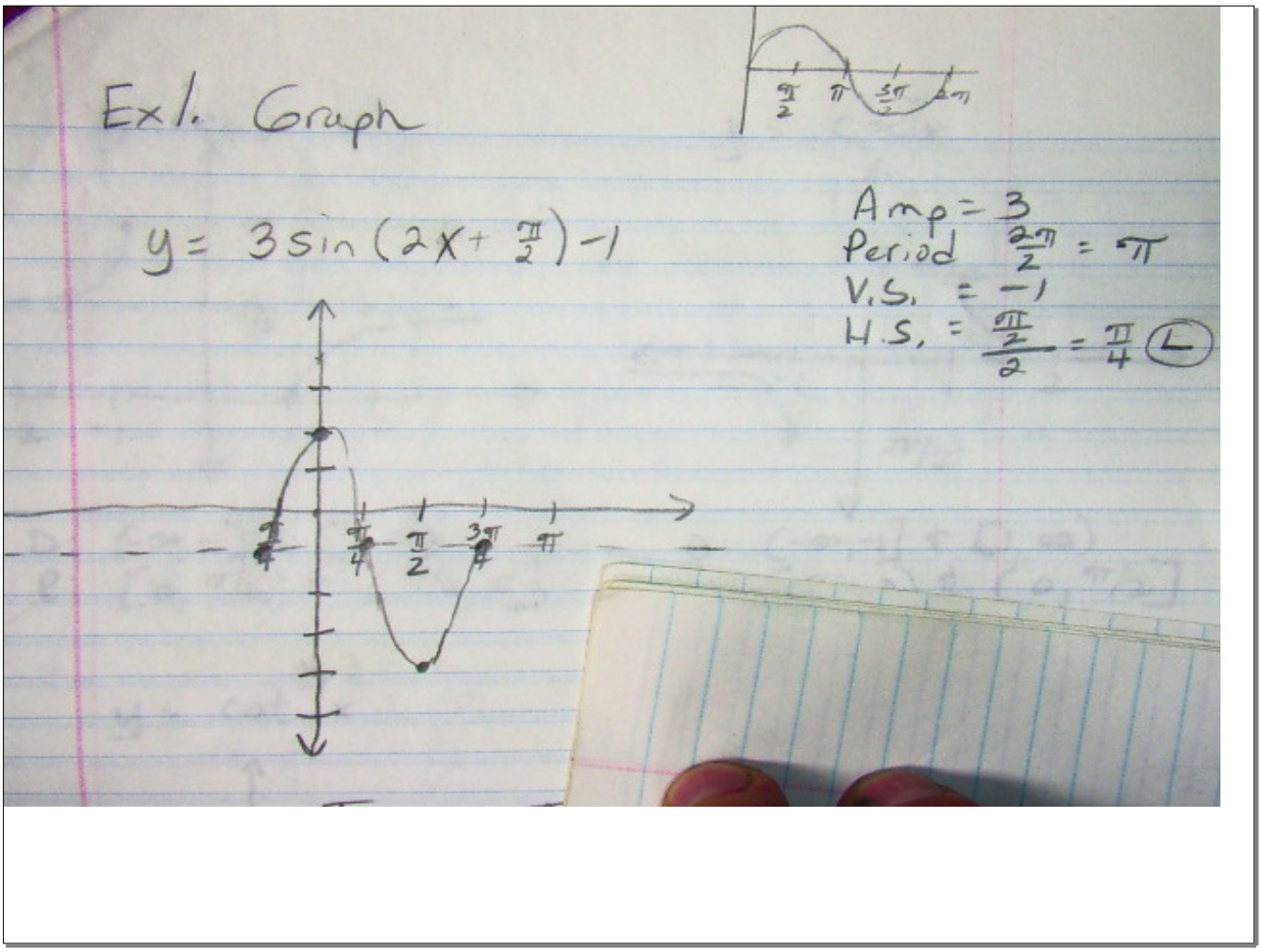
Ex 1. Graph
y = 3 sin (2x + π/2) − 1
Amp = 3
Period 2π/2 = π
V.S. = −1
H.S. = (π/2)/2 = π/4 (L)
π/2
π
5π/2
3π
π/4
π/2
3π/4
π

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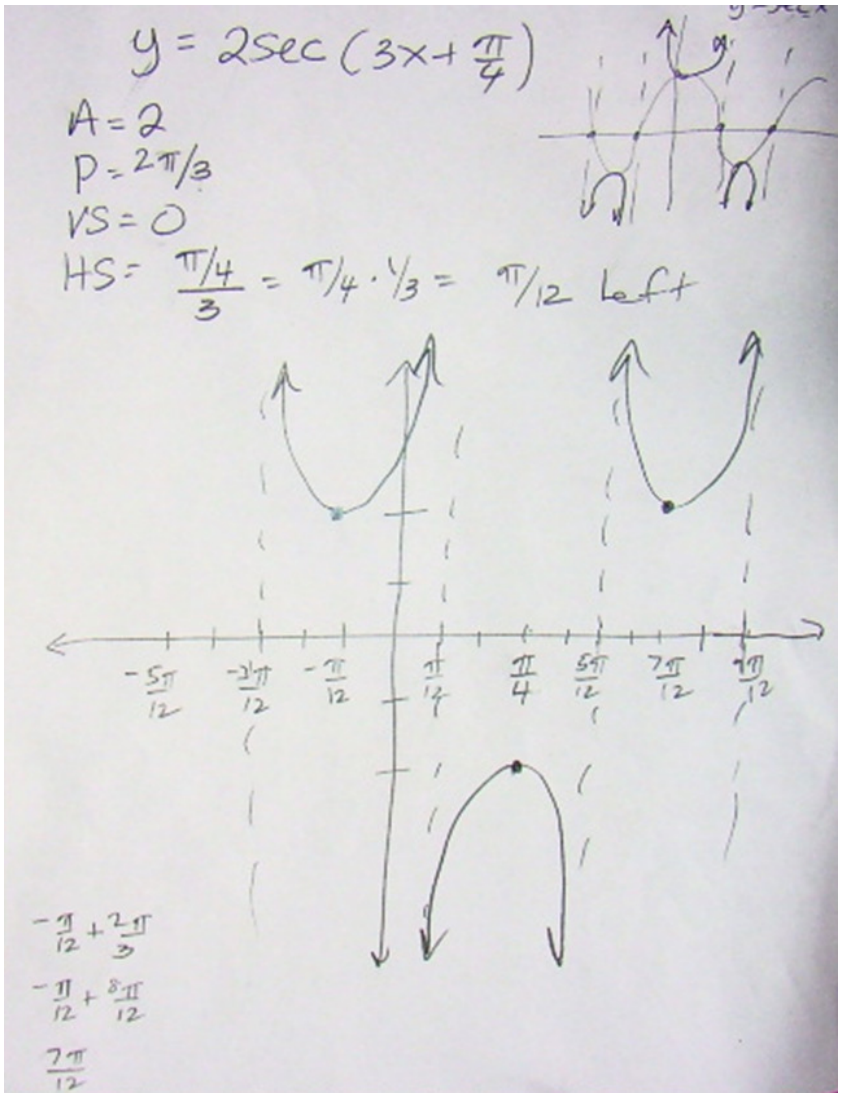
y = 2sec(3x + π/4)
A = 2
P = 2π/3
VS = 0
HS = π/4 / 3 = π/4 · 1/3 = π/12 Left
-5π/12
-3π/12
-π/12
π/12
π/4
5π/12
7π/12
11π/12
-π/12 + 2π/3
-π/12 + 8π/12
7π/12

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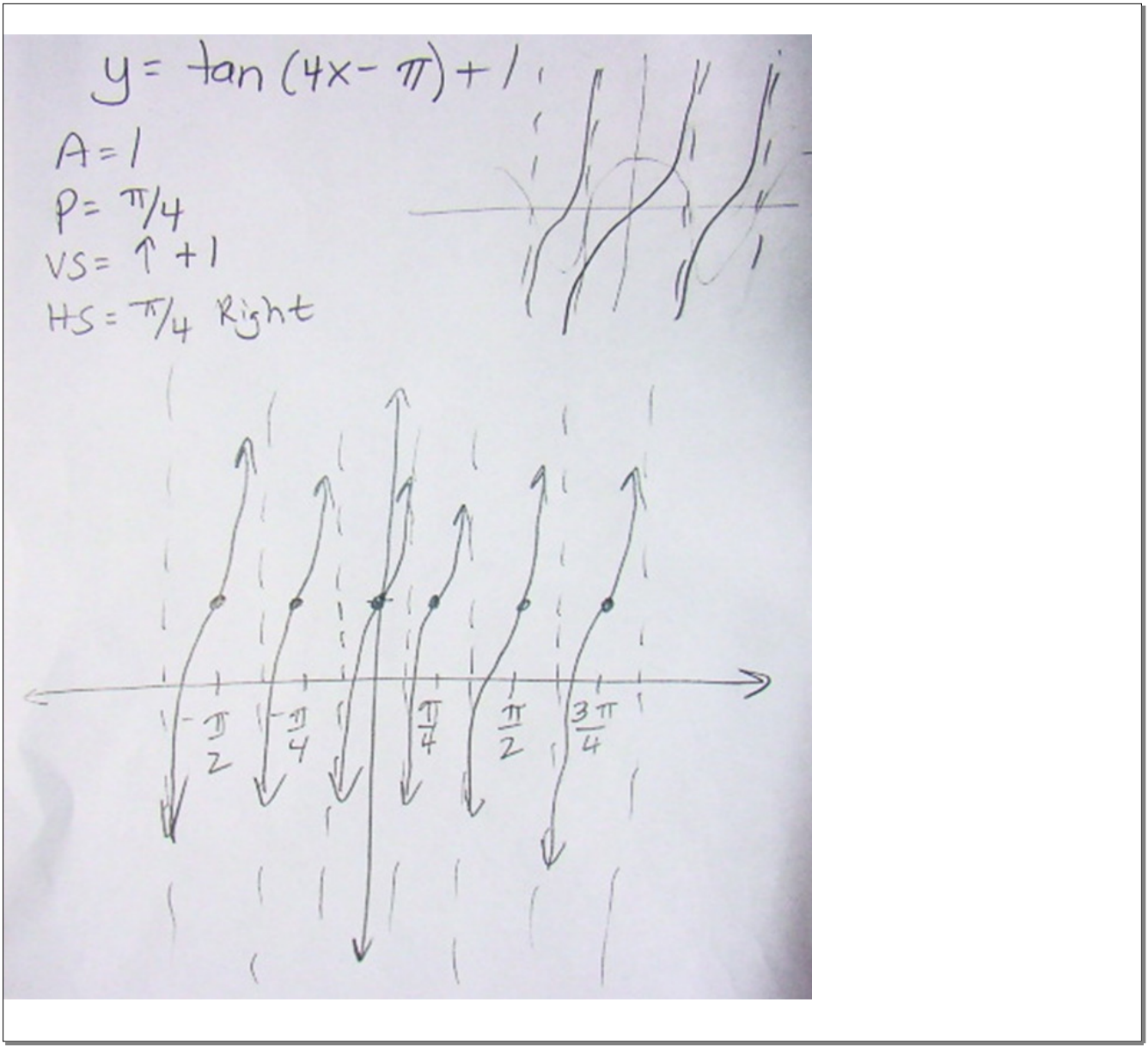

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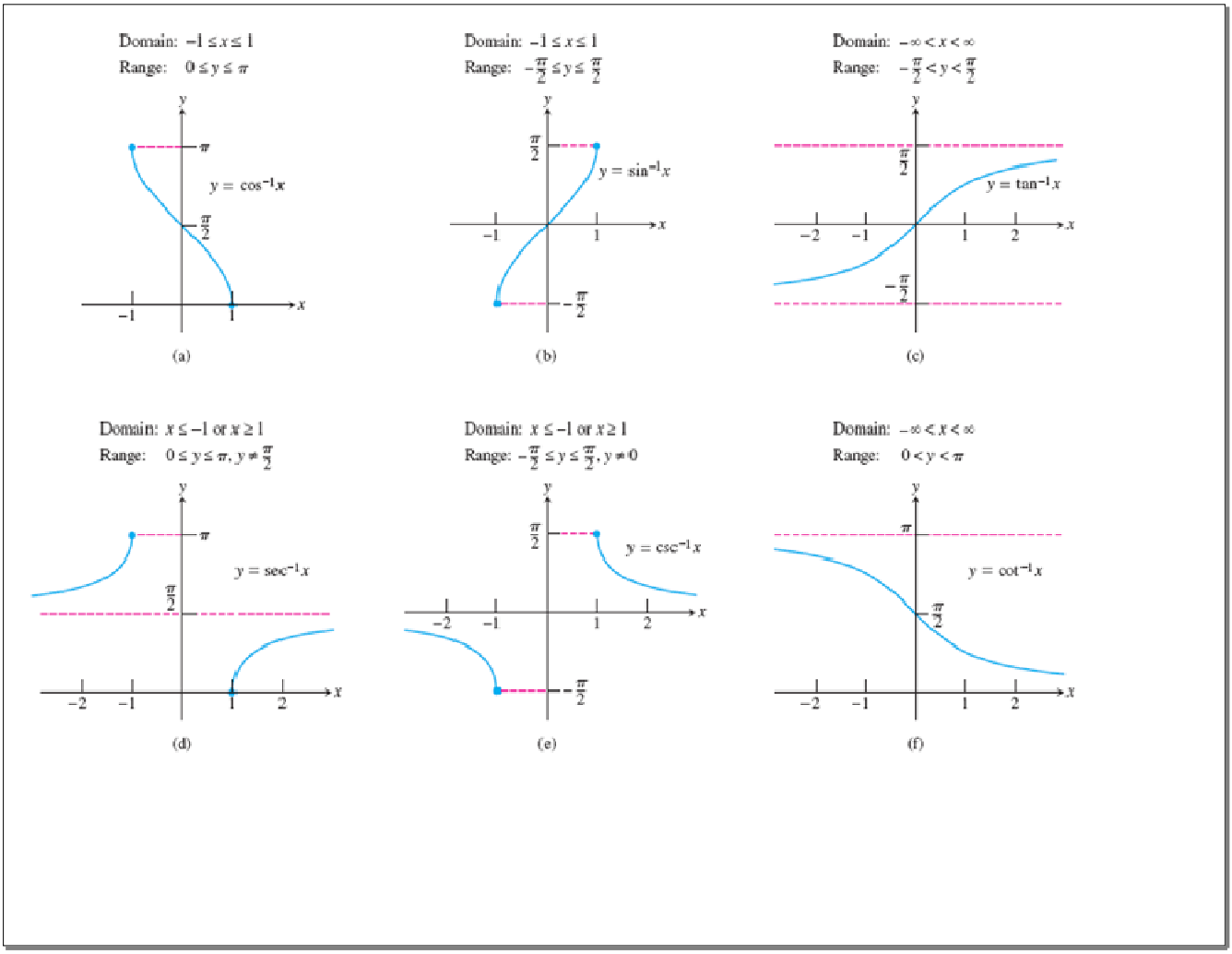
Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$
$y = \cos^{-1}x$
(a)
Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \sin^{-1}x$
(b)
Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \tan^{-1}x$
(c)
Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi, y \ne \frac{\pi}{2}$
$y = \sec^{-1}x$
(d)
Domain: $x \le -1$ or $x \ge 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \csc^{-1}x$
(e)
Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$
$y = \cot^{-1}x$
(f)

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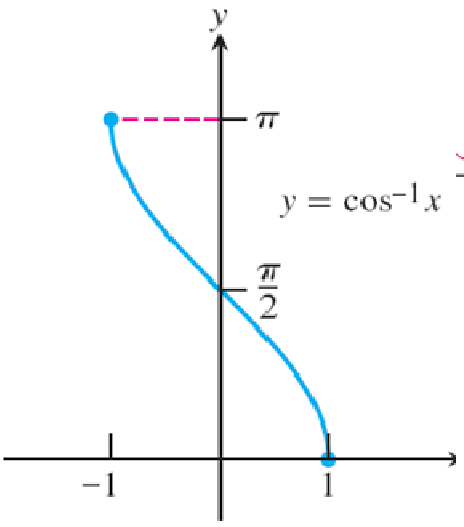
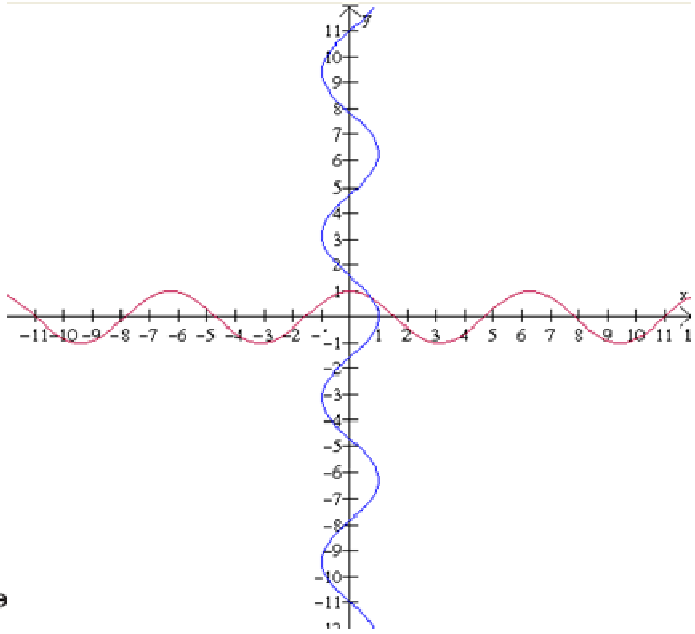
Domain: $-1 \le x \le 1$

Range: $0 \le y \le \pi$





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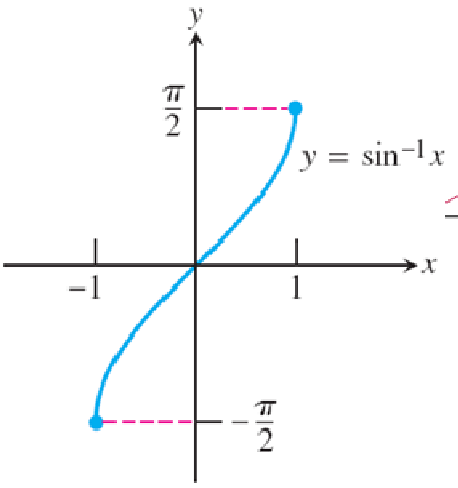
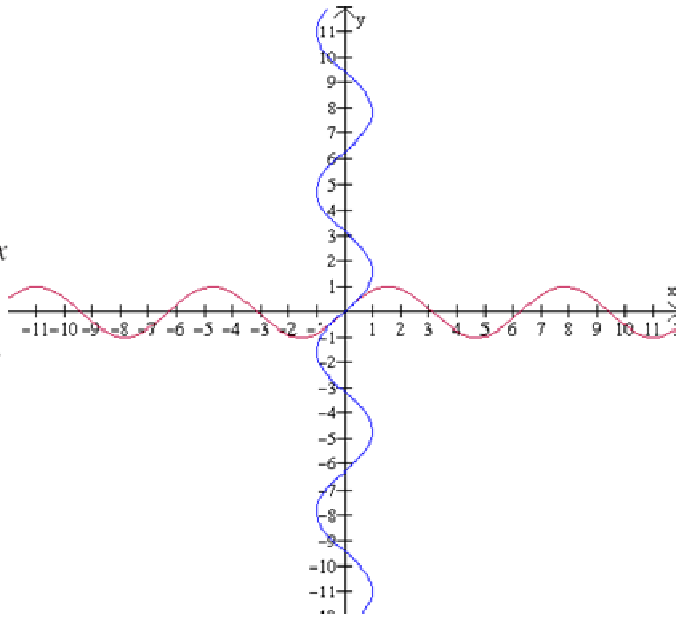
Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$





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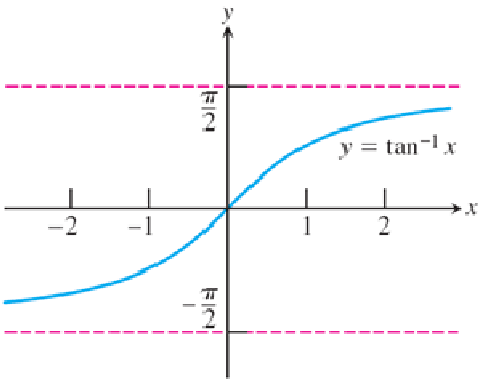
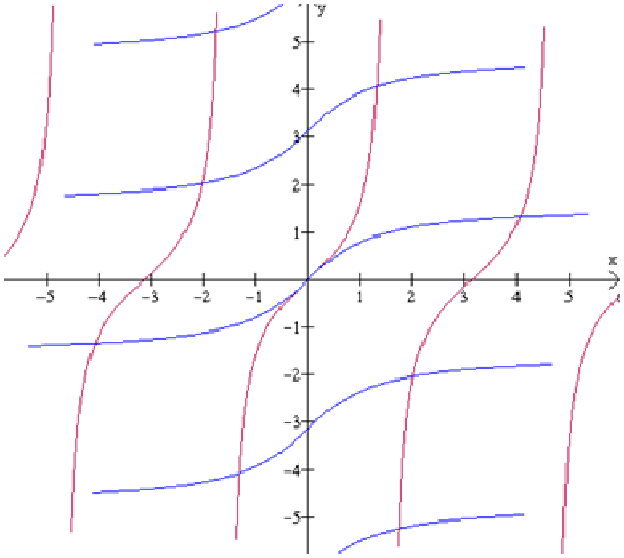
Domain: $-\infty < x < \infty$

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$





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Ex2. Solve for the specified interval

1.) $2\cos x - \sqrt{3} = 0$ all values of x

2.) $\sqrt{3}\tan x - 3 = 0$ $0 \le x \le 2\pi$

3.) $\sqrt{2}\sin x + 2 = 1$ $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

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$2\cos x - \sqrt{3} = 0$ all values of x

$\frac{2\cos x}{2} = \frac{\sqrt{3}}{2}$

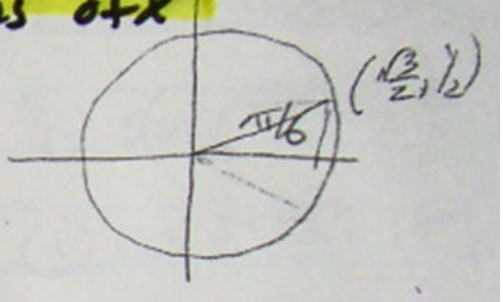
$\cos x = \frac{\sqrt{3}}{2}$

$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = \cos^{-1}(\sqrt{3}/2)$

$x = \pi/6 + 2\pi n$

$= 11\pi/6 + 2\pi n$



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$\sqrt{3}\tan x - 3 = 0$

$0 \leq x \leq 2\pi$

$3\tan x = 3$

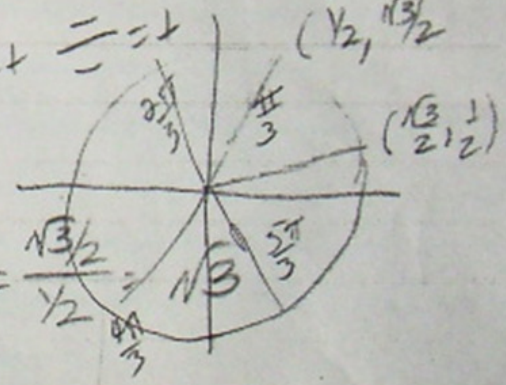
$\tan x = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$x = \tan^{-1}(\sqrt{3})$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

$\frac{+}{+} = +$ $\frac{-}{-} = +$

$\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$



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$\sqrt{2}\sin x + 2 = 1$

$-2 \quad -2$

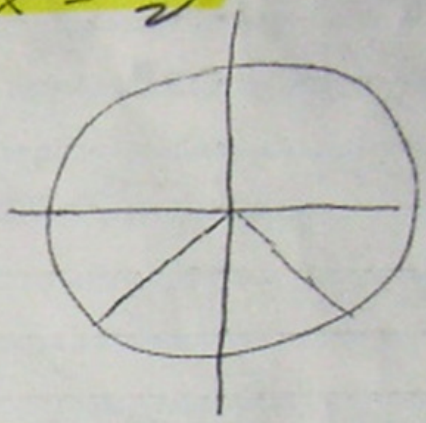
$\frac{\sqrt{2}\sin x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\sin x = -\frac{\sqrt{2}}{2}$

$x = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



$x = -\frac{\pi}{4}$

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$3\sin x - 1 = 0$

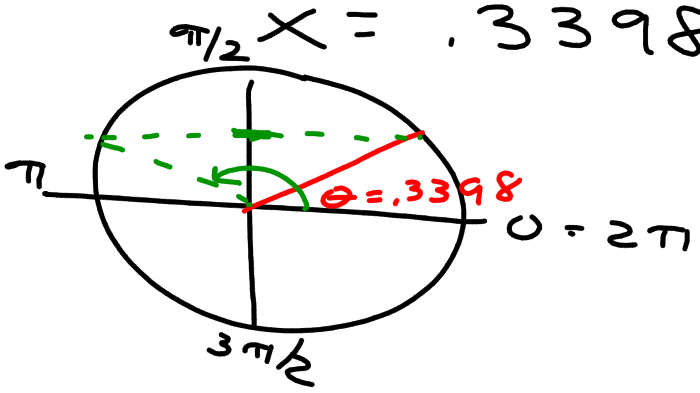
$3\sin x = 1$

$\sin x = 1/3$

$x = \sin^{-1}(1/3)$

$x = .3398$

$\sin^{-1}(1/3)$
$.3398369095$



$x = \pi - .3398$

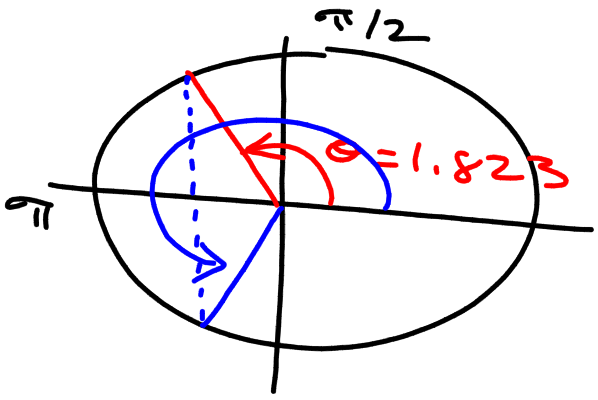
$x = .3398 \pm 2\pi n$
$= 2.802 \pm 2\pi n$

Sep 9-2:07 PM

$4\cos x + 1 = 0$

$\cos x = -\frac{1}{4}$

$x = \cos^{-1}(-1/4)$ $x = 1.823$



cos⁻¹(-1/4)
1.823476582

$2\pi - Ans$

$x = 4.460$

Homework

pg 52 # 7, 12, 13, 16, 18, 24, 31-36, 52-55

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